PERGAMON

International Journal of Solids and Structures 39 (2002) 601–616

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

The influence of matrix yield stress gradient on the growth and coalescence of spheroidal voids

Zhenhuan Li *, Cheng Wang, Chuanyao Chen

Department of Mechanics, Huazhong University of Science and Technology, Wuhan 430074, China

Received 1 November 2000; received in revised form 24 March 2001

Abstract

In a mass of engineering materials and structures, graded variations in yield stress is inherent. To investigate the macro-mechanical response and meso-mechanism of damage by void growth and coalescence in the materials with graded yield stress, detailed finite element computations of a representative cylindrical cell containing a spherical void are performed. By comparison with the responses of a homogeneous material cell model, significant effects of the matrix yield stress gradient (YSG) on the void growth and coalescence are revealed: (1) The evolution of voids in the homogeneous matrix materials has no close relations with the matrix yield stress level, however, the YSG distribution in matrix can lead to faster void growth rate and lower void coalescence strain. (2) In the homogeneous materials, the critical shapes of voids are independent on the yield stress of matrix materials, but the graded distribution of yield stress in the matrix materials, which leads to higher local plastic strain in the softer matrix layer, have an important influence on the critical void shapes. (3) The critical void volume fraction f_c is insensitive to the yield stress of the homogeneous matrix, however the YSG in matrix materials has a stronger effect on f_c, especially when the stress triaxiality level is lower. (4) Higher strain energy stored in the softer material layer encircling voids is internal driving force to faster void growth. The ratio of strain energy stored in the softer matrix layer surrounding voids to the whole energy provided by outside environment can give better description to the meso-mechanism of void evolution.

Keywords: Yield stress gradient; Porous material; Stress triaxiality; Coalescence; Cell model

1. Introduction

In most engineering structures consisting of multi-phase materials, composites and gradient materials, yield stress variations are inherent (Kolednik, 2000). Large numbers of experiments and computational studies have evidentially shown that the gradient in yield stress has very visible influences on the behavior of cracks (Suresh et al., 1992, 1993; Sugimura et al., 1995; Kim et al., 1997, 1999). The mechanics and meso-mechanisms of fracture and damage in the materials with graded yield stress, such as function gradient materials (FGM) and advanced composites, are the topics of considerable interests in the design of

* Corresponding author. Fax: +86-27-8754-3501.
E-mail address: zhli68@public.wh.hb.cn (Z. Li).

PII: S0020-7683(01)00098-1

structural components. Reliability concerns of some special structural components, such as soldered joints, nitrided or case hardened components, coating systems also call for a better understanding to the effect of material plasticity gradient on fracture resistance.

As everyone knows, the mechanics and meso-mechanism of microvoid evolution plays a key role in ductile fracture of plastically deformed solids. Similar with the fracture and damage processes in the homogeneous materials, the ductile fracture in the materials with graded yield stress can be divided into three phases, namely nucleation, growth and coalescence of micro-voids. With the plastic deformation increasing, voids are first nucleated at an inclusion or second phase via local interface decohesion or particle fracture, and then the growth of voids is firmly controlled by plastic straining of graded materials in the vicinity of voids until local internal necking and plastic collapse of the intervoid ligament occur. The latter event corresponds to the initiation of ductile fracture. Comparison with the plastic strain mode in the homogeneous materials, under similar exterior loading condition, the plastic deformation localization is accelerated seriously by the gradient distributions of yield stress in the matrix materials. Accordingly, the influences of yield stress gradient (YSG) in the matrix materials on the void growth and coalescence cannot be ignored.

In the past decades, the theoretic analyses of void evolution mechanism have been given a great deal of attentions. McClintock (1968) and Rice and Tracey (1969) investigated respectively the growth of an isolated cylindrical void and spherical void embedded in an infinite perfectly plastic solid subjected to remote triaxial stresses. On the basis of the R–T model, Huang (1991) gave a modification to the R–T model by more precise theoretic analyses on an infinite perfectly plastic solid containing a spherical void. To consider the interaction of adjacent voids, Gurson further considered a representative cell containing a spherical void and suggested a plastic potential function for porosity materials (Gurson, 1977). Since this model has the advantages of characterizing damage and fracture in ductile materials, considerable attentions have been paid to it. To describe the drop-off of stress carrying capacity in materials after voids coalescence, Tvergaard and Needleman (1984) made an important modification to the Gurson model. Their constitutive relation has been referred to as GTN model and several of its extensions have been used to predicate ductile damage and fracture in metal materials. The beauty of aforesaid models consists in revealing clearly the key role of the stress triaxiality and the effective plastic strain to the void growth and coalescence.

Years afterward, in order to improve agreement of aforesaid models with computational studies and experimental observations, a mass of theoretical researches and precise FE computational analyses have been done to probe into the basic reasons of these discrepancies. Considering the original Gurson model was derived from an approximate solution for rigid–perfectly plastic material cell containing a centered ideal spherical void. Two possible ways to improve it are to add the strain hardening effects of matrix materials and the void shape effects into the damage model. Tvergaard (1981, 1982) first carried out finite element computational analyses on the elastic–plastic matrix containing a doubly periodic array of cylindrical voids and an array of spherical voids in three directions, and tried to modify Gurson model by introducing two adjustable parameters q_1 and q_2 to reflect the interaction of voids and the strain hardening effect in the matrix. By detailed numerical studies on a power law viscous matrix material containing spherical voids, Duva (1986) and Leblond et al. (1994), compared his cell model with the Gurson's model and Tvergaard's model and found that in the higher triaxial stress fields, Gurson's model and his cell model were in fair agreement, but Tvergaard's model appeared to be overly compliant (Ma and Kishimoto, 1998). In the meantime, Becker et al. (1989), Tvergaard and Hutchinson (1993), Søvik and Thaulow (1997), Li and Kuang (2000) and Yee and Mear (1996) investigated the shape effect on the void growth in finite cells and infinite matrix materials, respectively. Based on the train of thoughts of Gurson, Gologanu et al. (1993, 1994) adopted a two-field approach, which consists of the incompressible shape-changing field and the expansion field analogous to electrostatics, to arrive at a plastic potential of porous material containing non-spherical voids. Newly, Pardoen and Hutchinson (2000) integrated Gologanu's model and Thomason's model (Thomason, 1985a,b) into an enhance model to consider the void shape effect and the onset of void coalescence, and each of these has been extended heuristically to account for strain hardening effect. In addition, other some effects, such as the plastic flow localization due to non-uniform void distribution (Onho and Hutchinson, 1984; Becker and Smelser, 1994) and the different void cluster size effect (Benson, 1995); void instability in the elastic–plastic solid (Huang, 1991; Tvergaard et al., 1992; Ashby et al., 1989); the three-dimension effect (McMeeking and Hom, 1990; Worswick and Pick, 1990; Nagaki et al., 1993; Richelsen and Tvergaard, 1994; Zhang and Zheng, 1997; Zhang et al., 1999; Kuna and Sun, 1996), the strain mode effect in matrix (Li et al., 2001) and bimaterial plasticity mismatch effect (Li and Guo, 2001) on the void growth and coalescence have also been studied deeply and widely. All analyses have shown clearly that, besides the stress triaxiality and effective plastic strain, there be other driving forces affecting the growth and coalescence of voids (Li et al., 2001).

Compared with mass of researches into the mechanics and meso-mechanisms of ductile fracture by void growth and coalescence in the homogeneous materials, little attention has been devoted to the meso-mechanisms of ductile fracture in the inhomogeneous materials, such as gradient materials with variational yield stress, under triaxial stress conditions. As be well known, in the plasticity gradient material (PGM), the matrix material layer possessing different plastic properties surrounds the voids. Due to different plasticity gradient distributions, the ratio between the strain energy stored in the inner layer of matrix and that in the whole matrix will be different in the given triaxial stress field. Therefore, for the PGM, we can roughly speculate that the plasticity gradient distribution of the matrix materials will have significant effects on the growth and coalescence mechanisms of voids. However, to our knowledge, no systematic analyses on the void growth and coalescence in the PGM have been made in the existing literatures. It is also susceptible whether the classical meso-mechanical model driven from the homogeneous porosity material can rationally predict the growth and coalescence of voids in PGMs.

These general backgrounds motivate our great interests to focus attention on void evolution in the matrix materials with graded yield stress. Another indirect reason comes from the question puzzling us at all time if the plasticity strain gradient (SG) and local strain mode in ductile matrix materials have visible effect on the void growth mechanism.

2. Cell model

2.1. Model description

Because the cell model can rationally describe the internal relations between the macroscopic mechanical responses and the microstructures of materials, it has been widely used to simulate and study the behavior of porous solids (Koplik and Needleman, 1988; Brocks et al., 1995; Kuna and Sun, 1996; Søvik and Thaulow, 1997; and so on). Fig. 1 shows the geometry of the cylindrical cell containing one spherical void in the PGM.

For convenience, the rectangular coordinates (x, y, z) are adopted in this paper (see Fig. 1). The origin of the coordinate system is located at the center of cell. Obviously, the initial void volume fraction can be written as

$$f_0 = \frac{2a_0^3}{3R_0^2 H_0}, \tag{1}$$

where a_0 is the initial radius of the void, R_0 and H_0 are the initial radius and half of the height of the cell, respectively. In this investigation, no loss generality only $f_0 = 0.001$ is considered.

The current void volume fraction f is defined as the ratio of the total void volume to the cell volume. For elastic–plastic incompressible matrix materials, the void volume fraction can be computed by

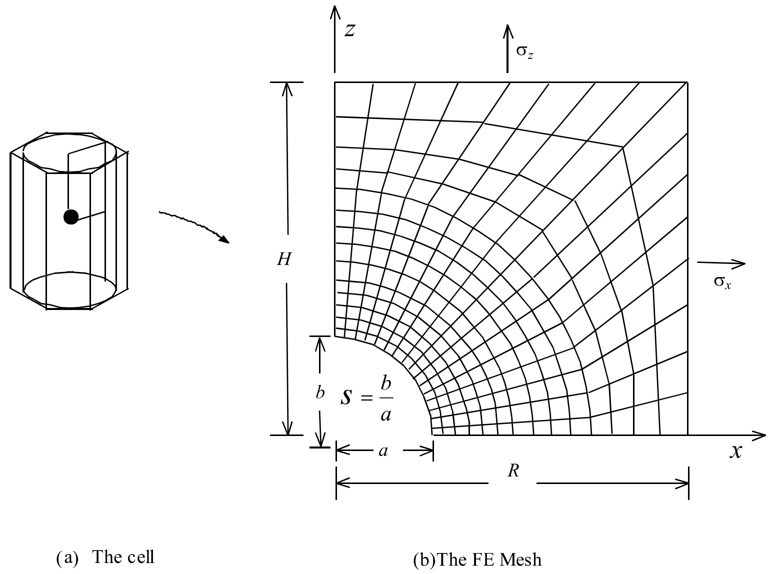

Fig. 1. The cell model.

$$f = 1 - \frac{(1-f_0)V_0 + \Delta V^{\mathrm{e}}}{V}, \tag{2}$$

where V_0 and V are the initial and current volume of the cell, respectively, ΔV^{e} is the increase in volume of the cylindrical cell due to elastic dilatation arising from the imposing hydrostatic stress which can be approximated by:

$$\Delta V^{\mathrm{e}} = V_0(1-f_0)\frac{3(1-2v)}{E}\sigma_{\mathrm{h}}, \tag{3}$$

where E and v are Young's modulus and Poisson's ratio of the matrix materials. σ_{h} is the macroscopic hydrostatic stress. Obviously, for the gradient matrix materials with identical elastic properties but different plastic properties, Eq. (3) can also be used to describe reasonably the elastic volume dilatation of the cell.

Considering the symmetry, quarter of the cell model (region $x_0 \geqslant 0$, $0 \leqslant z \leqslant H_0$) is analyzed with FEM. The finite element meshes of the geometry are constructed with 8-node quadratic iso-parametric element as shown in Fig. 1 and 2×2 Gauss integral are adopted. The axisymmetric FE cell model consists of 204 elements and 733 nodes as shown Fig. 1. To avoid the effect of FE mesh on the computational results, identical FE meshes are adopted in all FE analyses for the cell models with YSG and homogeneous matrix materials. The precision of the present FE computation has been validated be high enough to analyze the growth and coalescence of voids in the matrix materials (Li and Guo, 2001).

2.2. Boundary conditions and loading method

Due to the axial symmetry, the shear stresses on x, z-plane will vanish. Therefore, the axial, radial and tangential directions will be the principal directions of stresses. Furthermore, for axial symmetry case the tangential and radial components of the macroscopic stress and strain tensors subjected on the cell boundary will be of equal magnitude. The axisymmetric cell is subjected to a homogeneous elongation,

$U_{z\infty}$, in the axial direction and the radial displacement $U_{x\infty}$ is also kept to be homogeneous by constraint conditions.

The loads subjected on the axisymmetric cell are the prescribed axial deformation $U_{z\infty}$ and radial traction $F_{x\infty}$, so the boundary conditions for the axisymmetric region analyzed numerically can be described as:

$$\begin{aligned}
&\dot{u}_z = 0, \quad \dot{F}_x = 0; \quad \dot{F}_y = 0, \quad \text{on } z = 0, \\
&\dot{F}_x = \dot{F}_y = \dot{F}_z = 0, \quad \text{on } x^2 + y^2 + z^2 = a^2, \\
&\dot{u}_z = \dot{U}_{z\infty}, \quad \dot{F}_x = 0, \quad \dot{F}_y = 0, \quad \text{on } z = H, \\
&\dot{F}_x = \dot{F}_{x\infty}, \quad \dot{F}_y = 0, \quad \dot{F}_z = 0, \quad \text{on } x = R.
\end{aligned}$$

Here a is the current radius of the void, R and H are the current radius and half of the height of the cell, respectively; $\dot{u}_z$ is the axial displacement rate; F_i $(i = x, y, z)$ is the traction in the ith direction.

The prescribed radial traction $F_{x\infty}$ is determined from the condition that the average macroscopic true stresses acting on the cell follow the proportional history

$$\frac{\sigma_x}{\sigma_z} = \frac{\dot{\sigma}_x}{\dot{\sigma}_z} = \rho \tag{4}$$

with ρ is a prescribed constant determined by the controlled stress triaxiality R_σ and

$$\sigma_x = \sigma_y = \frac{F_x}{2\pi(R_0 + U_x)(H_0 + U_z)}, \tag{5}$$

$$\sigma_z = \frac{F_z}{\pi(R_0 + U_x)^2}, \tag{6}$$

where σ_i $(i = x, y, z)$ is the macroscopic principal stress in the ith direction.

For the axisymmetric case, the corresponding macroscopic effective stress and hydrostatic stress can be given by:

$$\sigma_{\mathrm{e}} = |\sigma_z - \sigma_x|; \quad \sigma_{\mathrm{h}} = \tfrac{1}{3}(\sigma_z + 2\sigma_x) \tag{7}$$

and the stress triaxiality R_σ can be written as

$$R_\sigma = \frac{\sigma_{\mathrm{h}}}{\sigma_{\mathrm{e}}} = \frac{1 + 2\rho}{3(1 - \rho)}. \tag{8}$$

The macroscopic principle strains and effective strain can be expressed by:

$$\varepsilon_x = \varepsilon_y = \ln\left(\frac{R}{R_0}\right); \quad \varepsilon_z = \ln\left(\frac{H}{H_0}\right); \quad \varepsilon_{\mathrm{e}} = \frac{2}{3}\left|\varepsilon_x - \varepsilon_y\right|. \tag{9}$$

In order to control the macroscopic stress triaxiality R_σ throughout the loading history, ρ has to remain constant whereas the ratio of the prescribed strains $\varepsilon_x/\varepsilon_z$ will vary with time. Similar to method suggested by Søvik and Thaulow (1997), to satisfy this requirement, a spring element, which measures the axial traction F_z arising from the prescribed axial displacement $U_{z\infty}$, is introduced. The radial traction $F_{x\infty}$ depending on R_σ is then applied. This loading process can be achieved automatically in the FE computation by the equilibrium iteration technology.

2.3. Matrix material description

In this work, the Prandtl–Ruess's constitutive equation is adopted to describe elastic–plastic stress vs. strain responses of the matrix materials. In the case of large deformation, the Prandtl–Ruess's constitutive equation can be written as:

$$\dot{\bar{\sigma}}_{ij}^{\mathrm{J}} = \frac{E}{1+v}\left[\delta_{ik}\delta_{jl} + \frac{v}{1-2v}\delta_{ij}\delta_{kl} - \lambda\frac{\bar{\sigma}'_{ij}\bar{\sigma}'_{kl}}{\frac{2}{3}\bar{\sigma}_{\mathrm{e}}^2\left(1+\frac{2}{3}h\frac{1+v}{E}\right)}\right]\dot{\bar{D}}_{kl}, \tag{10}$$

where $\dot{\bar{D}}_{ij}$ is the rate of deformation tensor, and $\dot{\bar{D}}_{ij} = 1/2\left(\partial v_i/\partial x_j + \partial v_j/\partial x_i\right)$, v_i is the rate of material point, x_i is the instantaneous coordinate of material point; $\dot{\bar{\sigma}}_{ij}^{\mathrm{J}}$ is the Jaumann rate of Cauchy stress $\bar{\sigma}_{ij}$; δ_{ij} is the Kronecker delta; h is the strain hardening parameter; $\bar{\sigma}'_{ij}$ is the deviatoric stress tensor defined by $\bar{\sigma}'_{ij} = \bar{\sigma}_{ij} - (1/3)\delta_{ij}\bar{\sigma}_{kk}$; $\bar{\sigma}_{\mathrm{e}}$ is the effective stress of the matrix; λ is the loading coefficient determined by the loading criteria:

$$\lambda = \begin{cases} 0 & \bar{\sigma}'_{ij}\dot{\bar{D}}_{ij} < 0, \\ 1 & \bar{\sigma}'_{ij}\dot{\bar{D}}_{ij} \geqslant 0. \end{cases} \tag{11}$$

The true stress vs. true strain responses of the matrix materials under uniaxial tension are assumed to be in the linear elastic–power hardening plastic form:

$$\bar{\sigma} = \begin{pmatrix} E\bar{\varepsilon}, & \bar{\varepsilon} \leqslant \sigma_{\mathrm{ys}}/E, \\ \sigma_{\mathrm{ys}}\left(E\bar{\varepsilon}/\varepsilon_{\mathrm{ys}}\right)^n, & \bar{\varepsilon} > \sigma_{\mathrm{ys}}/E, \end{pmatrix} \tag{12}$$

where $\bar{\sigma}$ and $\bar{\varepsilon}$ are the uniaxial stress and strain, respectively, σ_{ys} and $\varepsilon_{\mathrm{ys}}$ are the uniaxial yield stress and yield strain, respectively, n is the strain hardening exponent. In this paper, $E = 200$ GPa and $n = 0.1$ are chosen.

The main purpose of this paper is to investigate how the YSG in the matrix material affects the void growth and coalescence, and try to explain why gradient in yield stress affects the void evolution. As be well known, using radially graded yield stress from the center of void to simulate the local chemical composition or residual stress or strain fluctuation is rather appropriate. For this end, it is supposed that the matrix materials consist of different material layers with different yield stresses. As everyone knows, the gradient distribution of yield stress in the matrix materials has close relation to the materials constitute and the specific metal process performed on the materials. No loss generality, five possible radial YSG in the matrix materials are assumed. Fig. 2 displays the gradient variations of yield stress σ_{ys} with the radial distance r from the center of void. When the yield stress σ_{ys} is a constant, the matrix is a homogeneous material; on

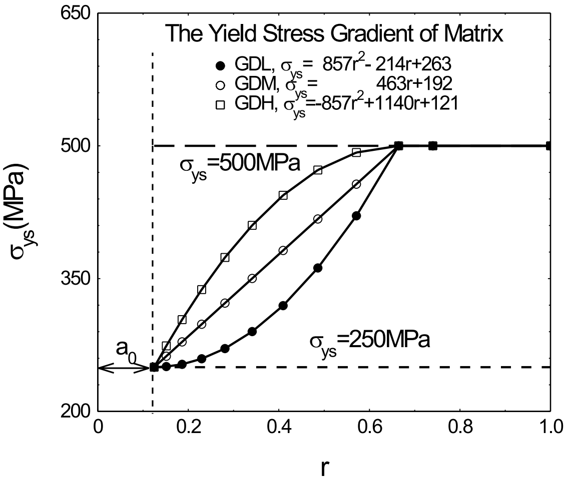

Fig. 2. The gradient distribution of yield stress in the matrix materials.

the contrary, the matrix is a PGM. Of course, the yield stress fluctuation or graded distribution zone encircling void is properly very smaller, smaller than the range in Fig. 2. However, as primary quantitative studies, the assumed graded distribution of yield stress in Fig. 2 still is appropriate.

3. Analyses of the computational results

As everyone knows, the stress triaxiality is the most important driving force to void evolution. In this investigation we will explore the dependences of the void growth and coalescence in the gradient materials on the macroscopic stress triaxiality. A range of stress triaxiality $0.75 \leqslant R_\sigma \leqslant 3.0$ will be considered, which includes necking smooth bars to sharp crack bodies (Koplik and Needleman, 1988).

3.1. The macroscopic response of cell model

To obtain deeper understanding for meso-mechanical mechanism of porous material, it is very necessary to analyses the macro-mechanical behavior and its response of cell models. Fig. 3 shows the computational effective mechanical responses of the cell model for the material with YSG GDH under four different triaxial stress fields, namely stress triaxiality $R_\sigma = 0.75$, $R_\sigma = 1.0$, $R_\sigma = 1.5$ and $R_\sigma = 3.0$. Fig. 3(a) displays the macroscopic effective stress–effective strain curves. With deformation increasing, the effective strain increases gradually, and then the effective stress decreases slowly, finally the abrupt fall in the strain–stress curve which clearly reflects the break down of the load carrying capability of the cell caused by void growth and internal necking comes forth. Fig. 3(b) gives the changes in cell radius against the effective strain. It is shown clearly that the plastic collapse is eventually reached, at which the effective strain increases while the radial strain remains approximately constant. This event, which marks sudden shift to a uniaxial straining mode in the intervoid radial ligament, corresponds to the void coalescence. As can be seen in Fig. 3(c) and (d), at this point the void volume fraction increases steeply and the intervoid ligament width $2D$ $(D = R - a)$ decreases rapidly. Therefore, alike the homogeneous materials (Li and Kuang, 2000; Li and Guo, 2001), it is reasonable to define the point, where the macroscopic radial strain keeps a constant with increasing macroscopic effective strain, as the inception of void coalescence (Koplik and Needleman, 1988; Søvik and Thaulow, 1997).

3.2. Influence of the matrix yield stress gradient on the void evolution

As be widely known, the macroscopic stress triaxiality and effective plastic strain have been regarded as two important driving forces controlling void growth at all times. In past 30 years, large numbers of attentions is fixed on the homogeneous matrix materials; no systemic analyses on the void growth and coalescence in the inhomogeneous matrix materials have been made. To probe into the influence of YSG in the matrix materials on the void evolution, in this section, the cell models with different matrix YSG (which is denoted by materials with lower gradient yield stress (GDL), materials with middle gradient yield stress (GDM), and materials with higher gradient yield stress (GDH), in Fig. 2) will be considered. Fig. 4 shows the macroscopic stress–strain responses of the homogeneous material cell as well as the material cell with graded yield stress under four typical stress triaxiality levels. For two homogeneous matrix materials with different yield stresses, although the carrying capacities are different, the critical coalescence strains $(\varepsilon_e)_c$ are almost identical due to same strain hardening exponent n. As be expected, for the matrix materials with graded yield stress, until the attainment of the critical constant radial strain, the response curves of the cells lie between the curves of the two kinds of homogeneous material cells. However, it is surprising that the critical coalescence effective strains $(\varepsilon_e)_c$ of the gradient materials are remarkably lower than that of both homogeneous matrix materials, and the lower the YSG near the void is, the lower the critical coalescence

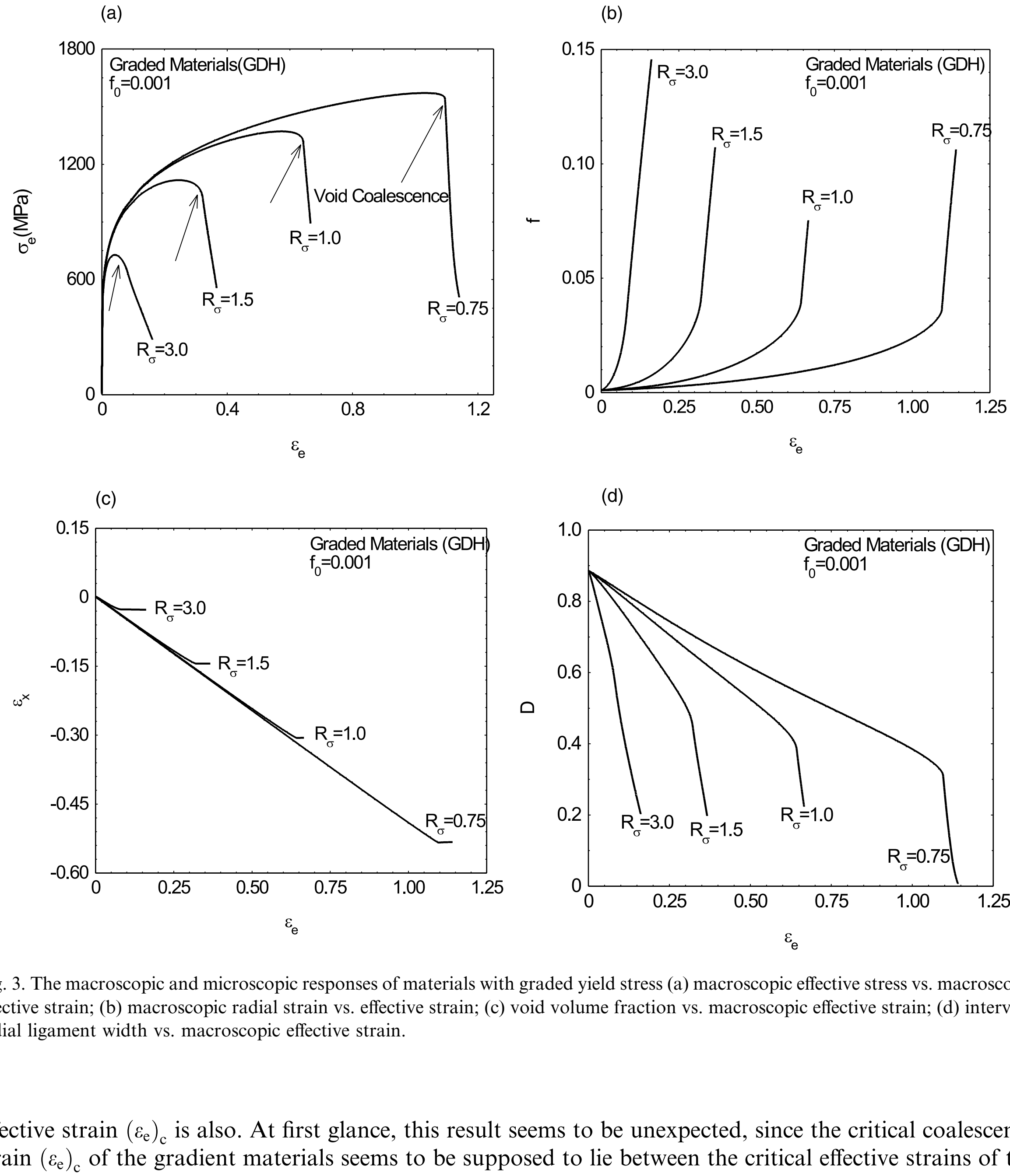

Fig. 3. The macroscopic and microscopic responses of materials with graded yield stress (a) macroscopic effective stress vs. macroscopic effective strain; (b) macroscopic radial strain vs. effective strain; (c) void volume fraction vs. macroscopic effective strain; (d) intervoid radial ligament width vs. macroscopic effective strain.

effective strain $(\varepsilon_e)_c$ is also. At first glance, this result seems to be unexpected, since the critical coalescence strain $(\varepsilon_e)_c$ of the gradient materials seems to be supposed to lie between the critical effective strains of the two kinds homogeneous materials with same strain hardening exponent but different yield stress. Again, the actual explanation lies in the ratio of plastic strain energy stored in the softer material layer encircling voids to the whole energy provided by outside environment is increased markedly due to the YSG distribution.

Fig. 5 compares the void growth in the homogeneous materials with that in the graded matrix materials for different stress triaxiality levels. It is easy to find that, for two homogeneous materials with different

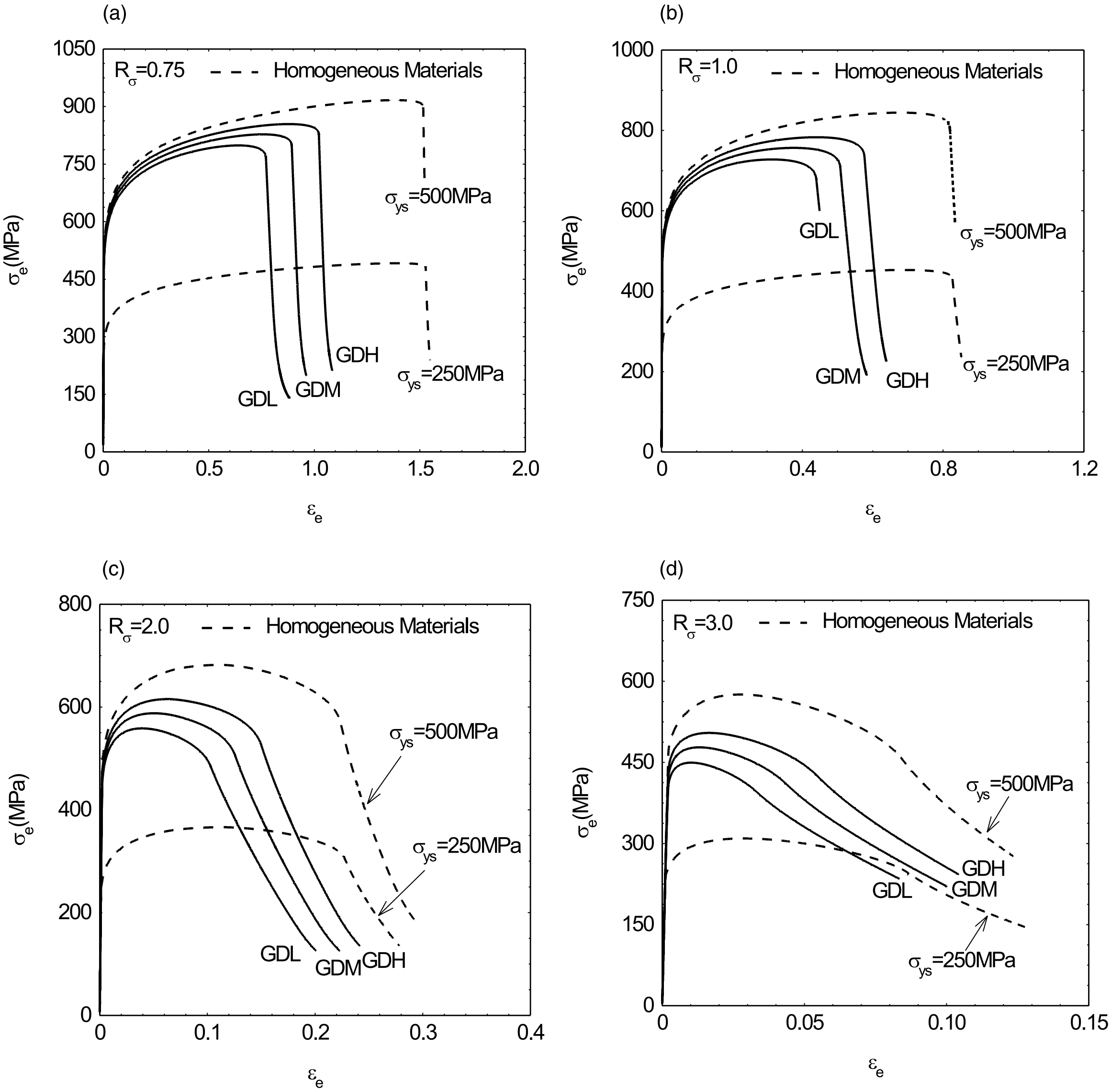

Fig. 4. The influence of yield stress gradient in the matrix materials on the macroscopic stress–strain curves for (a) $R_\sigma = 0.75$; (b) $R_\sigma = 1.0$; (c) $R_\sigma = 0$; (d) $R_\sigma = 3.0$.

yield stresses, the growth rates of voids are almost identical. This makes clear that, for the homogeneous materials, the evolution of voids is insensitive to the yield stress of matrix materials. By this token, we appear to be able to speculate that the YSG in the matrix materials has less effect on the growth and coalescence mechanisms of voids. However, from Fig. 5, we can clearly see that, the growth rates of voids in the matrix materials with graded yield stress are much faster than that of voids in the homogeneous materials. Therefore, the effect of the YSG distributions on the void growth is very strong. This means that the damages in the matrix materials with graded yield stress are much more dangerous than that in the homogeneous matrix materials under same loading condition.

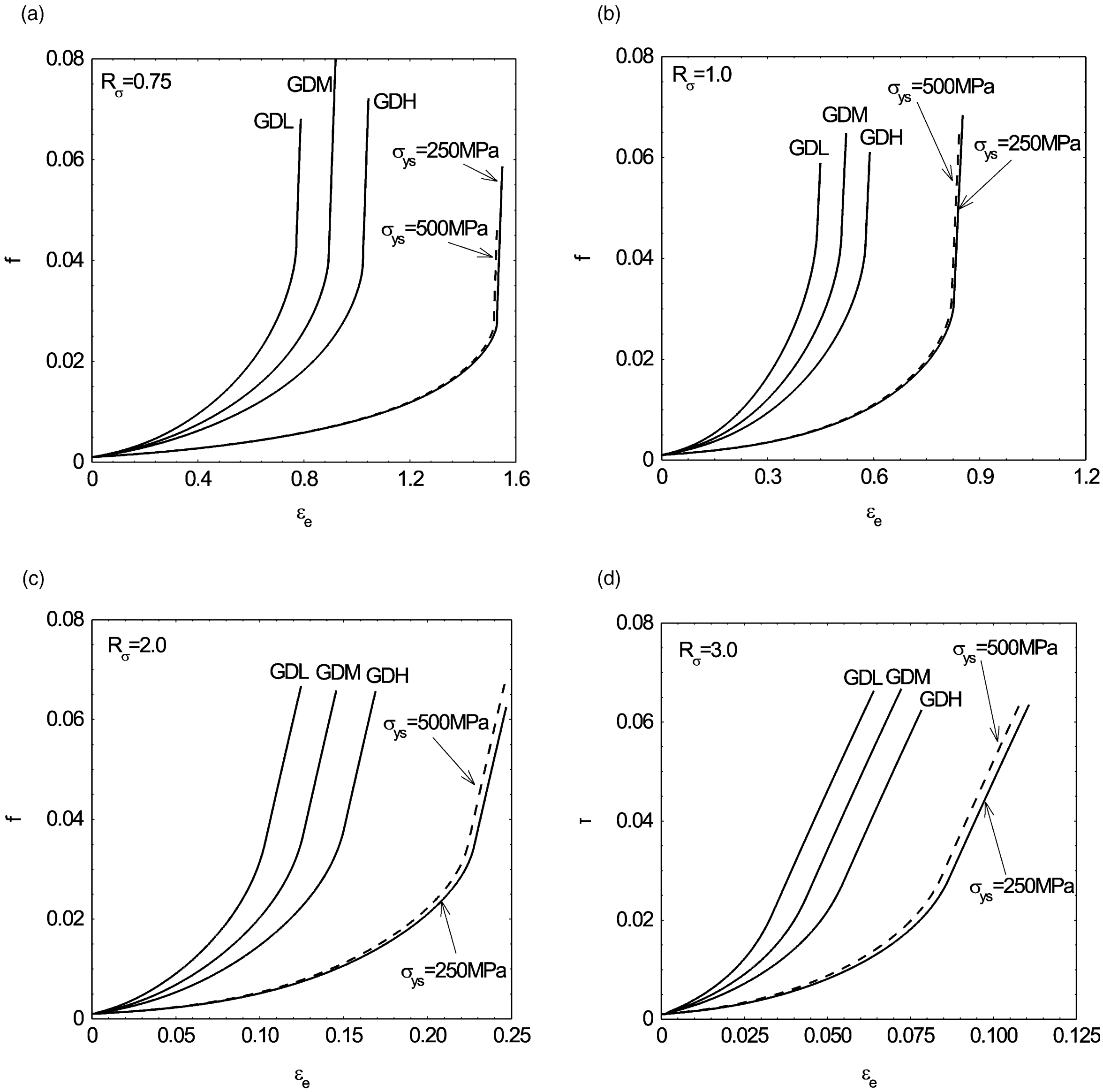

Fig. 5. The influence of yield stress gradient in the matrix materials on the void growth for (a) $R_\sigma = 0.75$; (b) $R_\sigma = 1.0$; (c) $R_\sigma = 2.0$; (d) $R_\sigma = 3.0$.

Fig. 6 shows the variations of the critical coalescence effective strain $(\varepsilon_e)_c$ with the stress triaxiality R_σ for the homogeneous matrix and the graded matrix materials. It can be found that, due to the YSG distributions in the matrix materials, the coalescences of adjacent voids happen earlier, so the critical strain $(\varepsilon_e)_c$ of the gradient materials is much lower than that of the homogeneous materials.

Fig. 7 plots the critical void volume fraction f_c as a function of the stress triaxiality R_σ. From Fig. 7, it can been found that, in lower triaxial stress fields, f_c in the gradient matrix materials is much higher than that in the homogeneous matrix materials, but in the higher triaxial stress fields, f_c is nearly inde-

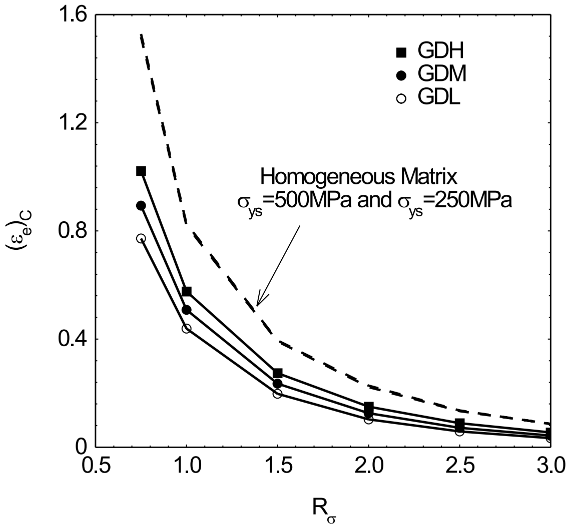

Fig. 6. The influence of yield stress gradient in the matrix materials on the void coalescence strain.

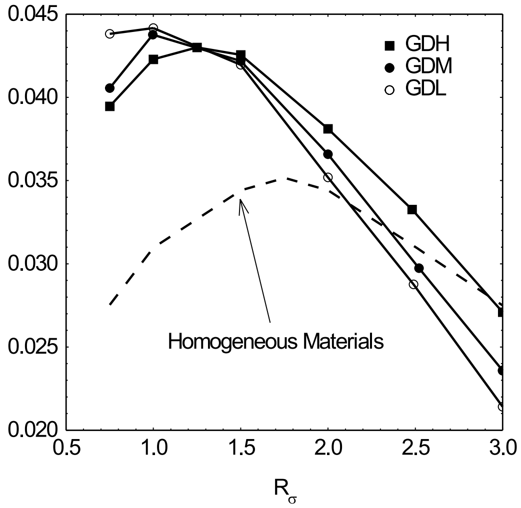

Fig. 7. The influence of yield stress gradient in the matrix materials on the void coalescence volume fraction.

pendent on the matrix gradient distributions. The explanation lies in that, in the lower triaxial stress fields, the shape change mechanism of void evolution is dominant in the homogeneous materials and the void growth is relatively difficult, but the volume change mechanism of void growth is dominant gradually with the yield stress gradient in the matrix materials increasing, as a result the void growth becomes relatively easier than that in the homogeneous materials; however, in higher triaxial stress fields, the volume change of void growth is main damage mechanism whether in the homogeneous or in gradient materials.

It is worthy of notice that f_c is sensitive to the stress triaxiality level for both the gradient and homogeneous materials. It is also susceptible whether f_c can be regarded as a material constant to predict fracture in specimens with different geometrical configurations although it was used to the damage criterion for a long time.

3.3. Meso-mechanism of void evolution in the graded material

The macroscopic stress triaxiality R_σ and the effective plastic strain ε_e^p are regarded as two important driving forces to the void evolution. However, the large plastic strains within the matrix materials layer surrounding voids play a key role to move the boundary layer of voids. Therefore, the meso-plastic strain modes in the vicinity of voids have internal relations with the volume change and the shape change of voids. As be pointed out by Li et al. (2001), the ratio of strain energy stored in the softer material layer encircling voids to the whole energy provided by outside environment can be regarded as a better measure of the effective energy driving void evolution, which can be defined:

$$R_w = \frac{\int_{\Omega_i} \frac{1}{V_i}\left(\int_0^{\varepsilon_e^{ip}} \frac{\sigma_{ij}^i}{\sigma_{ys}^i}\,d\varepsilon_{ij}^{ip}\right) dV^i}{\int_{\Omega_0} \frac{1}{V_0}\int_0^{\varepsilon_e^{0p}}\left(\frac{\sigma_{ij}^0}{\sigma_{ys}^0}\,d\varepsilon_{ij}^{0p}\right) dV^0} = \frac{\int_{\Omega_i} \frac{1}{V_i}\left(\int_0^{\varepsilon_e^{ip}} \frac{\sigma_e^i}{\sigma_{ys}^i}\,d\varepsilon_e^{ip}\right) dV^i}{\int_0^{\varepsilon_e^{0p}} \frac{\sigma_e^0}{\sigma_{ys}^0}\,d\varepsilon_e^0 + \frac{1}{3}\int_0^{\varepsilon_e^{0p}} \frac{\sigma_{kk}^0}{\sigma_{ys}^0}\,d\varepsilon_{kk}^{0p}}, \tag{13}$$

where σ_{ij}^i and ε_{ij}^{ip} is the stress and plastic strain tensors in the materials layer in the vicinity of voids respectively; σ_{ij}^0 and ε_{ij}^{0p} is the macroscopic stress and plastic strain tensors subjected on the cell boundary; σ_{ys}^i and σ_{ys}^0 is the yield stress of the materials located in the vicinity of the voids and on the cell boundary, respectively, which are introduced to normalize the stress level in the matrix materials; V_i is the characteristic volume of the materials layer surrounding voids, which has close relation with the characteristic scale in the materials, and V_0 is the volume of the whole cell, and $V_i = \int_{\Omega_i} dV^i$, $V_0 = \int_{\Omega_0} dV^0$.

Fig. 8 shows the variations of R_w with the macroscopic effective strain ε_e. With deformation increasing, the plastic strain zone enlarges toward the cell boundary and the plastic strain energy stored in the matrix materials turns to more and more uniform, so R_w decreases gradually with increasing effective strain ε_e. From Fig. 8, we can see that the R_w–ε_e curves have inherent relations with the macroscopic stress triaxiality R_σ. The higher the stress triaxiality R_σ is, the higher the R_w–ε_e curve is also, and the faster the rate of void growth is. It is striking to find that the R_w–ε_e curves are very sensitive to the YSG distributions in the matrix materials. For the homogeneous matrix materials, although there are larger differences in the yield stress of the matrix materials, the R_w–ε_e curves are almost coincident; on the contrary, for the gradient matrix materials, the R_w–ε_e curves are obviously higher than that of the corresponding homogeneous materials, and the lower the yield stress gradient near the voids is, the higher the R_w–ε_e curve is, therewith the faster the rate of void growth is and the earlier the void coalescence takes place.

In Fig. 9, comparisons of deformed void shapes in the graded matrix materials and in the homogeneous matrix materials are given for two given triaxial stress levels. It is surprising to find that, besides the stress triaxiality, the YSGs in the matrix materials cause also such significant differences in the deformed void shapes. For homogeneous materials, the effects of the matrix yield stress differences on the void shapes can be ignored; however, for the graded matrix materials, the critical shapes of voids are sensitive to the YSGs in the matrix materials, especially when the stress triaxiality is lower. It is worthy of noticing that, in the lower triaxial stress fields, comparisons with the deformed void shapes in the homogeneous matrix materials where the shape change mechanism of voids is dominant, the radial extend of voids in the graded matrix materials is larger. This means that though the shape change mechanism of void evolution is main in lower triaxial stress fields, the volume change mechanism of void growth in the graded materials cannot also be ignored, as a result the rates of void growth in the graded materials are much faster, and the

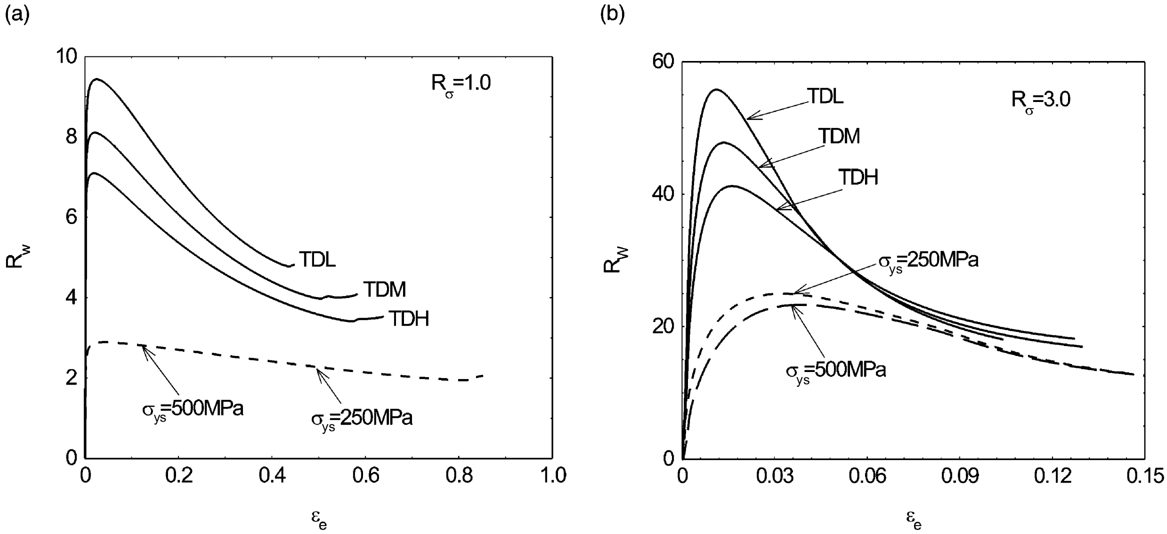

Fig. 8. The influence of yield stress gradient in the matrix materials on R_w for (a) $R_\sigma = 0.75$; (b) $R_\sigma = 3.0$.

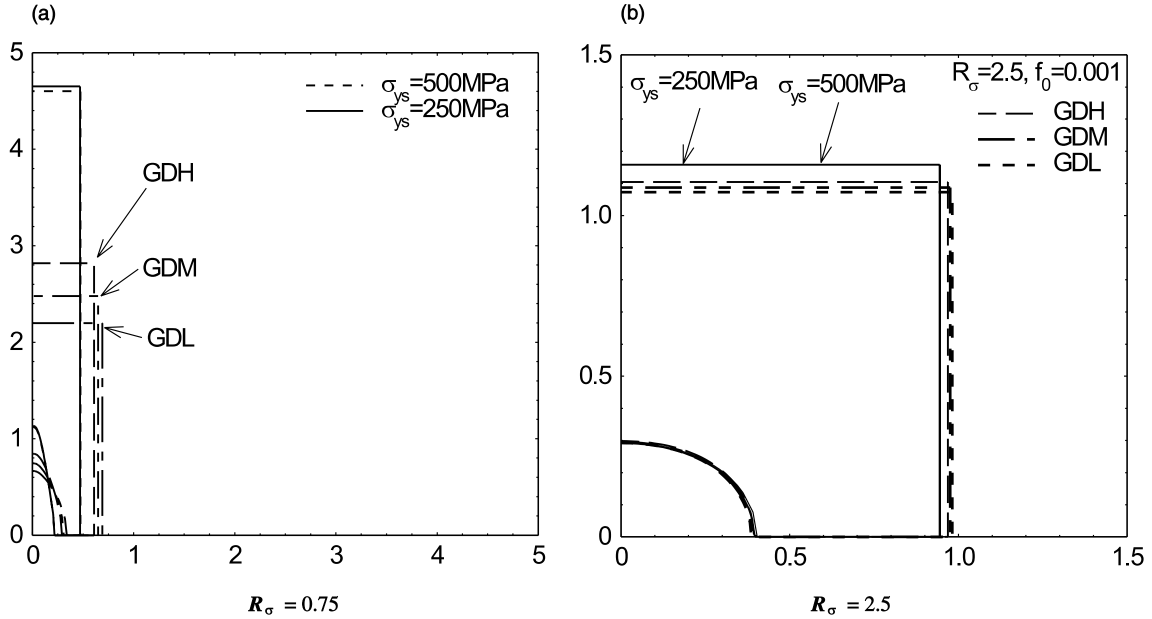

Fig. 9. The influence of yield stress gradient in the matrix materials on the void shape for (a) $R_\sigma = 0.75$; (b) $R_\sigma = 2.5$.

corresponding void coalescence takes place much earlier; in higher triaxial stress fields, due to the volume change mechanism of void growth in the homogeneous and gradient materials are dominating, the differences in critical void shapes are very small.

4. Discussions and conclusions

In the present paper, the effects of YSGs on the void growth and coalescence in the graded matrix materials have been investigated carefully by use of an axisymmetric cell model. From the numerical results the following conclusions relating to the YSG can be drawn:

- For the homogeneous matrix materials, the growth and coalescence of voids have not internal dependence to the yield stress of matrix materials, but are sensitive to the YSG in matrix materials.
- The critical effective strain $(\varepsilon_e)_c$ has close relation with the YSGs in the matrix materials. Comparison with the $(\varepsilon_e)_c$ of the homogeneous matrix materials, the $(\varepsilon_e)_c$ of the graded matrix materials decreases obviously, and the lower the YSGs are, the lower $(\varepsilon_e)_c$ is also. This means that the damages in the graded matrix materials are much more dangerous than that in the homogeneous matrix materials under identical exterior condition.
- The YSG distribution of matrix has a visible effect on the critical void volume fraction f_c. In the lower triaxial stress fields, the critical void volume fraction f_c of the graded materials is much higher than that of the homogeneous materials; however, in the higher triaxial stress fields, the effect of the yield stress gradients on f_c is relatively weaker.
- Besides the macroscopic stress triaxiality and effective plastic strain, the YSG in the matrix materials is another important factor affecting the critical void shapes. Under the lower triaxiality stress level, the aspect ratio of voids in the graded materials is smaller that in the homogeneous materials, and the lower the yield stress gradient in the vicinity of voids is, the smaller the aspect ratio of voids is, as a result the earlier the coalescence of void happens. However, under the higher triaxiality stress level, due to the volume change mechanism of void growth is main, the effects of yield stress gradient on the void shape become relatively weaker.
- Due to the YSG distribution in the matrix and the easier plastic flow localization in the softer material layer in the vicinity of voids, the higher strain energy stored in the material layer encircling voids can supply enough driving force to the void growing rapidly.

It is worthy of pointing out that, in lower triaxial stress fields, the voids in the homogeneous matrix materials are elongated along the axial direction and the growth of voids is very slow, so the shear mode of void growth and coalescence will be dominant; however, in the graded materials, due to higher local strain energy stored in the matrix surrounding voids, the internal necking mode of voids growth and coalescence cannot be ignored even under lower triaxiality levels, as a result the rate of void growth is obviously quickened.

As a byproduct, we obtained some new and interesting understanding about the inherent damage mechanism of the porous materials as follows:

The computational results have powerfully evidenced that the softer material layers surrounding the voids, which lead to higher local strain gradient, can drive the void growing faster. Accordingly, we can reason out that the harder material layers in the vicinity of the voids can markedly delay the voids evolving. These interesting phenomena put us in mind of paying more attentions to the YSG in the matrix materials encircling the voids rather than the whole yield stress level in the materials apart from the voids.

In the present paper, we use a YSG model to soften the materials layer in the vicinity of the voids, while Fleck and Hutchinson (1997) used SG mechanism to harden the matrix materials surrounding the voids. The implicit results by two different models are completely coincident. This hinted us that the PGM model may have a potentiality to bring void growth predictions into close agreement with the corresponding results of the SG or mechanism-based strain gradient (MSG) theories. Though a potential parameter R_w, which is the ratio of strain energy stored in the matrix layer in the vicinity of voids to the whole energy supplied by outside environment, has been suggested to characterize internal driving force to the void growth, the question if the parameter R_w can rationally characterize the scale effect in damage mechanism is still open. Further researches about R_w are in progress.

References

Ashby, M.F., Blunt, F.J., Bannister, M., 1989. Flow characteristics of highly constrained metal wires. Acta Metall. 37, 1857–1870.

Becker, R., Smelser, R.E., 1994. Simulation of strain localization and fracture between holes in an aluminum sheet. J. Mech. Phy. Solids, 42, 773–793.

Becker, R., Smelser, R.E., Richmond, O., Appleby, E.J., 1989. The effect of void shape on void growth and ductility in axisymmetric tension tests. Metall. Trans. A. 20A, 853–861.

Benson, D.J., 1995. The effect of void cluster size on ductile fracture. Int. J. Plasticity 11, 571–582.

Brocks, W., Sun, D.Z., Hönig, A., 1995. Verification of the transferability of micromechanical parameters by cell model calculations with visco-plastic materials. Int. J. Plasticity 11, 971–989.

Duva, J.M., 1986. A constitutive description of nonlinear materials containing voids. Mech. Mater. 5, 137–144.

Fleck, N.A., Hutchinson, J.W., 1997. Strain gradient plasticity. In: Hutchinson, J.W., Wu, T.Y. (Eds.), Advances in Applied Mechanics, vol. 33. Academia Press, New York, 295–361.

Gologanu, M., Leblond, J.B., Devaux, J., 1993. Approximate models for ductile metals containing non-spherical voids – case of axisymmetric prolate ellipsoidal cavities. J. Mech. Phy. Solids 41, 1723–1754.

Gologanu, M., Leblond, J.B., Devaux, J., 1994. Approximate models for ductile metals containing non-spherical voids – case of axisymmetric oblate ellipsoidal cavities. J. Engng. Mat. Technol. 116, 290–297.

Gurson, A.L., 1977. Continuum theory of ductile rupture by void nucleation and growth: Part I – Yield criteria and flow rules for porous ductile media. J. Engng. Mater. Technol. 99, 2–15.

Huang, Y., 1991. Accurate dilatation rates for spherical void in triaxial stress fields. J. Appl. Mech. 58, 1084–1086.

Kim, A.S., Besson, J., Pineau, A., 1999. Global and local approaches to fracture normal to interface. Int. J. Solids Struct. 36, 1845–1864.

Kim, A.S., Suresh, S., Shih, C.F., 1997. Plasticity effects on fracture normal to interfaces with homogeneous and graded compositions, Int. J. Solids Struct. 34, 3415–3432.

Kolednik, O., 2000. The yield stress gradient effect in inhomogeneous materials. Int. J. Solids Struct. 37, 781–808.

Koplik, J., Needleman, A., 1988. Void growth and coalescence in porous plastic solids. Int. J. Solids Struct. 24, 835–853.

Kuna, M., Sun, D.Z., 1996. Three-dimensional cell model analyses of void growth in ductile materials. Int. J. Fract. 81, 235–258.

Leblond, J.B., Perrin, G., Suquet, P., 1994. Exact results and approximate model for porous viscoplastic solid. Int. J. Plasticity 10, 213–235.

Li, G.C., Ling, X.W., Shen, H., 2001. On the mechanism of void growth and the effect of straining mode in ductile materials. Int. J. Plasticity 16, 39–58.

Li, Z.H., Kuang, Z.B., 2000. The mechanism and its effective energy criterion of void coalescence under different stress triaxiality. Acta Mech. Sinica 32, 428–438.

Li, Z.H., Guo, W.L., 2001. The influence of plasticity mismatch on the growth and coalescence of spheroidal voids. Int. J. Plasticity, in press.

Ma, F.S., Kishimoto, K., 1998. On yielding and deformation of porous plastic materials. Mech. Mater. 30, 55–68.

McClintock, F.A., 1968. A criterion of ductile fracture by growth of holes. ASME, J. Appl. Mech. 35, 363.

McMeeking, R.M., Hom, C.L., 1990. Finite element analysis of void growth in elastic–plastic materials. Int. J. Fract. 42, 1–9.

Nagaki, S., Goya, M., Sowerby, R., 1993. The influence of void distribution on the yielding of an elastic–plastic porous solid. Int. J. Plasticity 9, 199–212.

Onho, N., Hutchinson, J.W., 1984. Plastic flow localization due to non-uniform void distribution, J. Mech. Phy. Solids, 32, 63–85.

Pardoen, T., Hutchinson, J.W., 2000. An extended model for void growth and coalescence. J. Mech. Phys. Solids 48, 2467–2512.

Rice, J.R., Tracey, D.M., 1969. On the ductile enlargement of voids in triaxial stress fields. J. Mech. Phys. Solids 17, 201–207.

Richelsen, A.B., Tvergaard, V., 1994. Dilatant plasticity on upper bound estimates for porous ductile solid. Acta Metall. 42, 2561–2577.

Søvik, O., Thaulow, C., 1997. Growth of spherical voids in elastic–plastic solids. Fatigue Fract. Engng. Mater. Struct. 20, 1731–1744.

Sugimura, Y., Lim, P., Shih, C.F., Suresh, S., 1995. Fracture normal to a bimaterial interface effects of plasticity on crack-tip shielding amplification. Acta Metall. Mater. 43, 1157–1169.

Suresh, S., Sugimura, Y., Ogawa, T., 1993. Fatigue cracking in materials with brittle surface-coatings. Scripta Metall. Mater. 29, 237–242.

Suresh, S., Sugimura, Y., Tschegg, E.K., 1992. The growth of a fatigue crack approaching a perpendicularly oriented bimaterial interface. Scripta Metall. Mater. 27, 1189–1194.

Thomason, P.F., 1985a. Three-dimensional models for the plastic limit-load at incipient failure of intervoid matrix in ductile porous solids. Acta Metall. 33, 1079–1085.

Thomason, P.F., 1985b. A three-dimensional model for ductile fracture by the growth and coalescence of microvoids. Acta Metall. 33, 1087–1095.

Tvergaard, V., 1981. Influence of voids on shear band instabilities under plane strain conditions. Int. J. Fract. 17, 389–407.

Tvergaard, V., 1982. Ductile fracture by cavity nucleation between large voids. J. Mech. Phys. Solids 30, 265–286.

Tvergaard, V., Huang, Y., Hutchinson, J.W., 1992. Cavitation instabilities in a power hardening elastic–plastic solid. Eur. J. Mech. A/ Solids 11, 215–231.

Tvergaard, V., Hutchinson, J.W., 1993. Effect of initials void shape on the occurrence of cavitations instabilities in elastic–plastic of the solids, ASME, J. Appl. Mech-T, 60, 807–812.

Tvergaard, V., Needleman, A., 1984. Analysis of the cup–cone fracture in a round bar. Acta Metall. 32, 157–169.

Worswick, M.J., Pick, R.J., 1990. Void growth and constitutive softening in a periodically voided solid. J. Mech. Phys. Solids 38, 601–625.

Yee, K.C., Mear, M.E., 1996. Effect of void shape on the macroscopic response of non-linear porous solids. Int. J. Plasticity 12, 45–68.

Zhang, K.S., Bai, J.B., Francois, D., 1999. Ductile fracture of materials with high void volume fraction. Int. J. Solids Struct. 36, 3407–3425.

Zhang, K.S., Zheng, C.Q., 1997. 3D analysis of spherical void contained cell under different triaxial stress state. New Progress of Solid Mechanics, Tsinghua University Press.